Nonreciprocal Electromagnetic Wave Propagation in Ionized Gaseous Media*

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Summary—The nonreciprocal propagation of electromagnetic waves in ionized gaseous media is discussed, and experimental observations are reported in this paper. The classical Faraday experiment in the optics of anisotropic media has suggested an analogous phenomenon at microwave frequencies. The anisotropic behavior of the free electron gas which is immersed in a magnetic field and subjected to an incident electromagnetic wave is determined. Guided microwave experiments were performed which confirm the theoretical predictions of nonreciprocal wave propagation in such ionized gases.

Introduction

THIS PAPER is concerned with propagation of high radio frequency electromagnetic waves in ionized gases. More particularly, we shall be concerned here with those general restricted conditions of the ionized gaseous media under which such wave propagation is nonreciprocal.

Before describing the properties of such media, it appears advisable to review in broad terms the main historical aspects of wave propagation effects which led to the discovery of the conditions of nonreciprocity in em wave propagation. We are referring here to the phenomenon observed by Faraday (in 1845) in the course of his studies of the change in the properties of matter under the influence of magnetic fields. In these investigations Faraday used visible light as a probe. Since visible light was used to investigate the change in the bulk properties of matter, this had to be transparent to the light beam so that true propagation in the medium could take place. This requirement led, first, to the use of glass, a transparent dielectric. As is well known, the effect consists of the rotation of the plane of polarization of the plane polarized light waves that are transmitted through the substance when it is immersed in a longitudinal steady magnetic field. This rotation, in substances isotropic when not immersed in magnetic fields, was found to be maximum when the direction of the light wave propagation was parallel or antiparallel with the direction of magnetization.

In 1884, A. Kundt performed the Faraday type experiment by sending light waves through appropriate thicknesses of ferromagnetic metallic foils that were immersed in longitudinal magnetic field. The Faraday effect in paramagnetic substances was first observed in 1906 by J. Becquerel.

While the first observations were made with visible light, the phenomenon has later been extended to very broad regions of the em spectrum on both sides of the visible. Looking back at these original experiments, among the most significant facts appears to be that which concerns the magnitude of the measured rotations.[1]

The systematic experiments of Verdet with a large number of substances transparent to visible light, performed in 1854 and following years, have indicated, as a rule, the proportionality of the rotation angle with the magnetic field intensity and the total (optical) path of the medium traversed by the light waves.

$$\theta = V_e \cdot H \cdot L \cdot \cos \phi. \tag{1}$$

Here H is the magnetic field, L is the total light path, ϕ is the angle of H with the direction of wave propagation, and V_e is a constant of proportionality characteristic of the substance. This constant is known as the Verdet constant. This relation appears to be valid for all substances which are transparent to the light waves used. The Faraday-Verdet rule, however, loses its validity for light waves whose frequency is close to a proper frequency or absorption band of the substance.

It is another historical fact that the Faraday effect has not been satisfactorily interpreted until the discovery of the Zeeman effect some fifty years after the original Faraday experiments.

The interpretation of the Faraday effect, as is now well known, makes use of the Zeeman effect, and it is based on the Fresnel decomposition of a linearly polarized em wave into two circularly polarized waves where the Fresnel vectors are of equal amplitudes and which rotate in opposite senses, at the frequency of the wave. This mathematical "artifice" introduced by Fresnel was instrumental in interpreting the natural rotation phenomena of plane polarized light that was observed in birefringent crystals prior to Faraday's magnetic rotation discovery.

The oppositely rotating circularly polarized waves propagate in the medium with different velocities (phase and energy). If N_+ is the index of refraction of the wave rotating to the right of an observer looking at the source, and N_- is that of the wave which rotates to the left, while N is the index of refraction in the medium in the absence of the magnetic field, we have

$$N_+ = N - \frac{\theta}{L} \cdot \frac{c}{\omega} \quad \text{and} \quad N_- = N + \frac{\theta}{L} \cdot \frac{c}{\omega}$$

where c is the wave velocity in free space and ω is the angular frequency of the propagated wave.

[1] To give an order of magnitude of the rotation angle θ, in glass about 30,000 times as thick as the wavelength, the maximum rotation observed after one crossing was 12° in a field of several thousand Gauss.

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It follows from the above that the angle of rotation

$$\theta = L\cdot(N_- - N_+)\,\frac{\omega}{2c}\cdot \tag{2}$$

The problem of predicting the magnitude of the magnetic rotation of the plane of polarization of a linearly polarized em wave traversing a certain transparent medium is reduced to the determination of the refractive indices N_- and N_+, that is to say, of the velocities of the two oppositely rotating circular waves which compose it.

This assumes that the medium is equally transparent for these circularly polarized waves. Should the medium be dichroic, however, then the rotated wave becomes elliptical with its large axis in the rotated direction. The angle of rotation remains, in general, little or unaffected. Turning now to the medium of our immediate interest, we shall have to define first the conditions of its transparency for high radio frequency em waves.

The Ionized Gaseous Medium

In an ionized gas there are, of course, free charge carriers of both sign. We shall consider here only those gases in which the free negative charges are free electrons and not negative ions. Electrons in all velocity ranges remain free in the inert monatomic gases and only in certain inert molecular gases such as N_2. For simplicity, we shall consider only the monatomic rare gases. The reason for this is that ions, positive or negative, due to their heavy mass, are not capable of oscillations with any significant amplitude in high-frequency em fields; thus, they do not directly affect high-frequency wave propagation in ionized gaseous media; only electrons are influential in determining propagation.

The region of an ionized gas where the free electron density is equal or very nearly so to the density of the positive ions is called a plasma. We shall be referring, in what follows, to gaseous plasmas or to gaseous discharge plasmas. In the latter case, the plasma is produced in the gas by electric fields, dc or ac (high or low frequency).

The free charge densities in laboratory gaseous discharge plasmas, n_e and n_+, are, in general, very much smaller than the densities of the gas atoms N_g, $(n_e/N_g \leq 10^{-5})$.

A plasma can be considered as a mixture of gases. This mixture is composed of the gas of the free electrons, the gas of the positive ions, and that of the neutral atoms (a fraction of which can be partly in short or partly in long life excited states). For such a mixture we can write the classical laws of the gas mixtures and write, for instance, that the total pressure P_t is the sum of the partial pressures of the constituent gases: $P_t = p_g + p_e + p_+$, or in terms of the energy content in these gases we have

$$P_t = N_g k T_g + n_e k T_e + n_i k T_+$$

where T, with the appropriate subscripts, represents the "temperature" of these constituent gases respectively. This assumes that the particles in each of these gases have a Maxwellian distribution of velocity.

$$\left(\text{Then}\,\frac{Mv^2}{2} = \frac{3}{2}\,kT\right).$$

From a thermodynamic viewpoint, we have two essentially different cases of interest: an isothermal plasma where $T_e = T_+ = T_g = T$ or a nonisothermal plasma where $T_e \neq T_g$, and $T_e \neq T_+$.

The plasma constituents are in continuous interaction with each other. This interaction, with kinetic energy exchange, takes place through collisions which occur at a frequency generally designated by ν, with the appropriate subscript, ν_{e-m}, ν_{e-i}, ν_{e-e}.[2] There is a direct relation between the temperature of the constituent gases and the collision frequency of the interacting particles. The most important relations of immediate interest in gaseous discharge plasma in which electromagnetic wave propagation takes place are

$$\nu_{e-m} \cong Q\cdot N_g\cdot \bar{v}_e = Q\cdot N_g\left(\frac{3kT_e}{m}\right)^{1/2} \sim A\cdot T_e^{1/2} \tag{3}$$

if Q is independent of v_e, A is constant. The collision cross section Q is dependent on the nature of the gas.

$$\nu_{e-i} = 3.6\,\frac{n_i}{T_e^{3/2}}\log_n\left\{\frac{3.7\times 10^3}{n_i^{1/2}}\;T_e\left(\frac{2T_e\cdot T_i}{T_e+T_i}\right)^{1/2}\right\}$$
$$\sim B\cdot\frac{n_i}{T_e^{3/2}} \tag{4}$$

where B is a constant, since the log term is very slowly varying.

Thus

$$\nu = \nu_{e-n'} + \nu_{e-i} \cong AT_e^{1/2} + B\cdot\frac{n_i}{T_e^{3/2}}\cdot \tag{5}$$

The interaction between the constituent gases implies a tendency toward a (thermodynamic) temperature equilibrium which can be reached, unless one of the constituents is supplied, selectively, with some kinetic energy —either continuously or otherwise. For instance, if a plasma is produced and maintained by an electric field, then the charges gain energy from the electric field up to that steady-state value at which the energy gained from the field equals the energy transferred to the other constituents of the gas mixture. So in the steady state of "running" (dc or ac) gaseous discharges, the electron gas temperature is always higher than the temperatures of the other constituents and $T_e \gg T_+ \gtrsim T_g$.

If, however, we remove the electric field that maintains a gaseous discharge and thus abandon the plasma (to its fate), then the constituent gases can reach a temperature equilibrium. It is understood that since in

[2] The subscripts *e-m*, *e-i*, and *e-e* designate electron-molecule, electron-ion, and electron-electron, respectively.

such a plasma the free charges are being lost through various processes and are not being replaced, the abandoned plasma will decay.

In order to have a better understanding of a gaseous discharge plasma as a propagating medium of em waves, it is advisable to inquire into the nature of the electronic collisions with neutral gas atoms and with the other charged constituents. In the presence of electric or electromagnetic field, the electrons acquire kinetic energy between collisions, that is to say, during their free time (τ). This is the time the electrons spend on their free path (λ_e). The energy thus acquired is ordered kinetic energy. If and when this ordered motion is interfered with by a collision, the motion becomes disordered or thermalized, and a fraction of the electron kinetic energy is lost to the partner with which the collision took place. The fraction of the energy transferred to a gas molecule, in such a collision, depends upon whether the electron kinetic energy is adequate or inadequate to excite any of the possible quantized states of the colliding partner. If it is inadequate, the collision could only be elastic, and the fraction of the electron energy lost on the average is about $2m/M$ where M, m are the masses of the colliding partners, respectively. In the other case, the collision may result with a certain probability in the excitation of the atomic colliding partner. (Ionization is, of course, a particular state of excitation.) In that case, a large fraction of the electron kinetic energy is lost. It is then obvious that the nature of the gas (He, Ne, A, Kr, Xe) and the pressure etc., play essential roles in the behavior of such plasmatic media.

Behavior in Microwave Fields

Let us consider the high-frequency behavior of gaseous discharge plasmas. In order to describe the wave propagation properties of such a medium, we have to determine the polarizability, or dielectric coefficient, the conductivity, and the magnetic permeability of the plasma, and their behavior in high frequency em fields. As mentioned earlier, only the free electrons of the plasma which are capable of oscillating with full amplitude at high radio frequencies, affect microwave propagation.

The conductivity of a free electron gas in vacuum, in microwave fields of frequency ω, is imaginary $\sigma = j(ne^2/m\omega)$. The effective dielectric coefficient of free space containing a gas of free electrons is

$$\frac{\epsilon}{\epsilon_0} = 1 - \frac{n \cdot e^2}{\epsilon_0 m \cdot \omega^2}, \tag{6}$$

where ϵ_0 is the dielectric constant of free space.

In a plasma where the gas of free electrons interacts with the other plasma constituents, the conductivity is complex.

$$\sigma_c = \sigma_r + j\sigma_i \tag{7}$$

(the subscripts c, r, i designate complex, real, and imaginary, respectively).

The complex conductivity of a plasma in the absence of any magnetic field as calculated by Margenau[3] and Ginsburg,[3] is shown in Fig. 1.

$$\sigma_c = \frac{4}{3} \frac{e^2\lambda_e n_e}{\sqrt{2\pi m K T}} \left[K_2(x_1) - j x_1^{1/2} K_{3/2}(x_1)\right] \tag{8}$$

where

$$x_1 = \frac{m(\omega\lambda)^2}{2kT}$$

n_e = electron density,
m = electron mass,
e = electron charge,
λ_e = electron mean free path,
k = Boltzmann's constant,
T = temperature (°K).

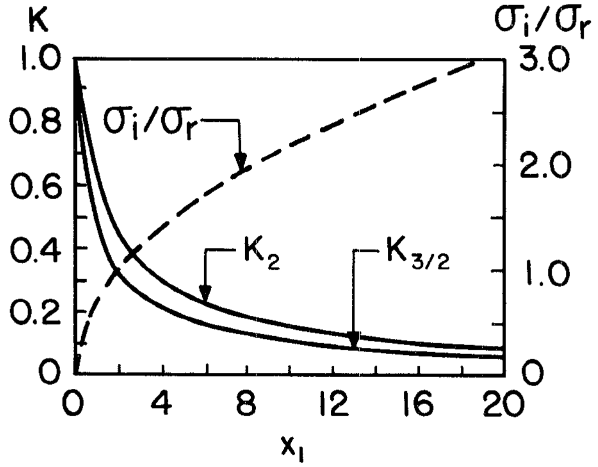


Fig. 1—Complex conductivity of a plasma (after Margenau[3]).

The functions $K_2(x_1)$ and $K_{3/2}(x_1)$ are plotted on Fig. 1. This formula of the complex conductivity of a gaseous plasma is approximated by the following simplified formula adequate for a general discussion:

$$\sigma_c = \frac{n_e e^2}{m}\left[\frac{\nu}{\nu^2 + \omega^2} - j\frac{\omega}{\nu^2 + \omega^2}\right] \tag{9}$$

where ν is the total collision frequency of the electrons. The magnetic permeability of a plasma of relatively low charge density is, essentially, that of free space in the absence of any magnetic field.

It is seen that both the real and imaginary parts of the complex conductivity are dependent on the electron density n_e and the electron collision frequency ν. The real part of the conductivity is maximum for $\nu = \omega$, in which case, $\sigma_r/\sigma_i = 1$.

At this point it is apparent that there are two plasma states of interest: 1) $\sigma_r/\sigma_i \ll 1$ and 2) $\sigma_r/\sigma_i \sim 1$. In the first case $\nu/\omega < 1$ and the plasma can be considered as a dielectric medium with a coefficient ϵ in the sense of electrostatics.

[3] H. Margenau, *Phys. Rev.*, vol. 69, p. 510; 1946.
V. Ginsburg, *J. Phys., U.S.S.R.*, vol. 8, p. 253; 1944. (In English.)

$$\frac{\epsilon}{\epsilon_0} = 1 - \frac{4\pi\sigma_i}{\omega} = 1 - \frac{4\pi n_e e^2}{m} \cdot \frac{1}{\nu + \omega^2} = 1 - \frac{\omega_p^2}{\nu^2 + \omega^2} \quad (10)$$

with

$$\frac{4\pi n_e e^2}{m} = \omega_p^2.$$

Whether in the absence of magnetic fields this dielectric plasma is transparent for em waves of frequency ω depends upon the sign of ϵ/ϵ_0. This is determined by the ratio of $\omega_p^2/\omega^2+\nu^2$.

The two extreme cases of interest mentioned from the viewpoint of ν and ω are 1) $\nu/\omega<1$ and 2) $\nu/\omega>1$. Case 1) indicates that the electrons are undergoing many oscillations per collision, which corresponds in general to low gas pressure and/or high frequencies, whereas case 2) indicates that the electrons undergo many collisions per oscillation. This case corresponds in general to high gas pressure and/or low-frequency oscillatory fields. In general, plasma conditions satisfying case 1) are selected for microwave frequency propagation in plasmas [although for certain cases; *e.g.*, case 2), could be more favorable]. In this case $\nu^2 \ll \omega^2$ and the real part of the conductivity is directly proportional to the electron density n_e and the electron collision frequency ν. The dielectric coefficient and, therefore, the wave propagation conditions, are controlled by the ratio of the plasma frequency to signal frequency. In particular, for frequencies lower than plasma frequency, no true propagation takes place since the phase velocity of the wave becomes imaginary.

$$V_{ph}^2 = \frac{c^2}{1 - \frac{\omega_p^2}{\omega^2}} \cdot \quad (11)$$

Under these conditions the plasma defines boundaries for wave propagation. This is the case of the ionosphere for the lower frequency ionospheric waves. However, these em waves still penetrate the plasma and progress in it with decreasing amplitude. The amplitude is decreased to $1/e$ of its value in a distance

$$d = \frac{c}{\omega_p}\left(1 - \frac{\omega^2}{\omega_p^2}\right)^{-1/2}$$

so that for $\omega \ll \omega_p$, d approaches $1/2\pi \times \lambda$ (free space) for radiation at the plasma frequency.

Since the plasma frequency is proportional to the square root of the electron density—a parameter essentially controllable—conditions for propagation of microwaves, or if desired, conditions for no propagation, can be adjusted at will. This control of wave propagation can be exercised on microsecond or shorter time scales, which makes this extremely flexible medium suitable for such wave propagation control.

Now for signal frequencies $\omega > \omega_p$ the wave propagates in the plasma. However, in view of the complex conductivity of a plasma in microwave (em) fields, its index of refraction is also a complex quantity.

$$\mathfrak{N} = N - jq,$$

where N is the ordinary index of refraction and q is the extinction index.

So that the amplitude of a wave of the form

$$E = a \cdot \exp\left[j\omega\left(t - \frac{\mathfrak{N}}{c} z\right)\right]$$

propagating in the z direction, that can also be written as

$$E = a \cdot \exp\left(-\frac{\omega}{c} qz\right) \times \exp\left[j\omega\left(t - \frac{N}{c} z\right)\right],$$

decreases as the wave progresses in the medium.

This attenuation is due to dissipation of em energy in the medium through the agency of the electrons. Under the above conditions of $\nu^2 \ll \omega^2$, attenuation is controlled by both n_e and ν.

Gaseous Discharge Plasmas in DC or Slowly Varying Magnetic Fields

In the absence of a magnetic field an unbounded plasma is isotropic for em wave propagation. The dispersion relation for such a plasma is given in (11), for negligible collisional losses. If we now place the plasma in a finite dc or slowly varying magnetic field, such that the field direction is parallel or antiparallel to the direction of em wave propagation, then the propagating properties of such a medium are modified in two essential ways:

1) The medium is no longer isotropic. It behaves as a uniaxial crystal. Indeed, propagation in any material medium is dependent upon the electric and magnetic polarizability of the medium. In the plasma, the magnetic polarization can be disregarded since the spin contribution to the permeability is largely negligible. The contribution of the orbital diamagnetic electron current loops can also be neglected, since, for conditions outside of the cyclotron resonance, the total diamagnetic moment of the Maxwell distributed free electron gas is zero. Neglecting losses, which in a way implies low level microwaves in practice, the electric induction $\vec{D} = \epsilon\vec{E} + \vec{P}$ will largely control the wave propagation in the plasma. Now, since the magnetic field applied in the z direction causes the electrons to describe circular orbits around the dc magnetic field, the high-frequency electric field of the wave, here normal to the dc magnetic field, will develop a component of motion at right angles to itself and in time quadrature with it.

This can be expressed in the following matrix representation:

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \epsilon & -j\epsilon' & 0 \\ j\epsilon' & \epsilon & 0 \\ 0 & 0 & \epsilon'' \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{12}$$

where we have assumed:

$$\frac{\epsilon}{\epsilon_0} = 1 + \frac{\omega_p^2}{\omega_H^2 - \omega^2}, \quad \frac{\epsilon}{\epsilon'} = \frac{\omega_H \cdot \omega_p^2}{\omega(\omega_H^2 - \omega^2)}; \quad \frac{\epsilon''}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega^2}.$$

$\omega_H = \gamma H_{DC}$ and γ = gyromagnetic ratio of electron. It is seen that in the gas plasma it is the dielectric constant which is a tensor as compared to the permeability tensor in the ferromagnetic substances.

2) The plasma frequency has lost direct control of the propagation of plane polarized em waves. The gyrofrequency of the electrons ω_H, now contributes to this control.

If the wave is plane polarized and thus composed of two oppositely rotating circularly polarized waves, it is rotated in its progression in the medium; the circularly polarized waves having different phase velocities. The dispersion relation—again for $\nu/\omega < 1$—becomes

$$V_{ph}^2 = \frac{c^2}{1 - \frac{\omega_p^2}{\omega^2} \cdot \frac{1}{1 \mp \frac{\omega_H}{\omega}}}. \tag{13}$$

The minus sign before ω_H applies if the electric vector of the wave rotates in the same sense as the gyrating electrons. This circular wave is the equivalent in optics to the extraordinary wave, in opposition with the other (or plus) wave called the ordinary wave.

The above relation can also be written as:

$$V_{ph}^2 = \frac{c^2}{1 - \frac{\omega_p^2}{(\omega')^2}}, \tag{14}$$

where

$$(\omega')^2 = \omega^2\left(1 \mp \frac{\omega_H}{\omega}\right).$$

This now shows more clearly whether propagation can take place depending upon the value of $\omega_p^2/(\omega')^2$.

Eq. (13) invites the following remarks. 1) For zero magnetic field we have (11) (that is $\omega' = \omega$). For zero plasma frequency, the propagation is that in vacuum. For infinite magnetic field, whatever the plasma frequency, it is the same as in vacuum.

2) The angle of rotation of the linear wave, observed at a distance L inside the plasma for each value of the magnetic field, depends upon the electron density in the plasma; this angle is being determined by the difference in the refractive indices of the two oppositely rotating circular waves, that compose the linearly polarized wave.

3) The angle of rotation varies rapidly in the vicinity of the plasma frequency, which is one of the proper frequencies of the magnetic field permeated plasma. This is, of course, also the case for $\omega \sim \omega_H$.

4) It is also apparent that for certain values of ω_H/ω and ω_p^2/ω^2 resonance conditions prevail. In addition to the electron cyclotron resonance, a plasma resonance can also be observed. This resonance is called the magneto-plasma resonance, since for any value of the dimensionless parameter of ω_p^2/ω^2, it is a function of the magnetic field intensity.

The discussion of the case of a loss-less unbounded infinite plasma, even though unrealistic, is extremely useful in that it points out the basic wave propagation properties of such a medium when used in any practical form.

Guided Wave Propagation in Plasmas

Guided microwave propagation through gaseous discharge plasmas is, in several respects, different from plane wave propagation in unbounded plasmas. However, since the basic properties of such a medium in a waveguide even though bounded remain largely unaffected, the propagation phenomena do not differ in their essential features from the unbounded case.

Here, wave propagation is, of course, dependent upon waveguide geometries, the waveguide modes. Phase velocity of the em waves (guide wavelength) and waveguide cutoff condition are important parameters of the problem.

The detailed theoretical aspects of Faraday rotation of guided waves through gyromagnetic media and to some extent gyromagnetic (gaseous) plasmas—outside of the scope of this paper—have been discussed in particular by Suhl and Walker[4] in this country and also by a group of French workers.[5] It does not appear to be fruitful to elaborate on these in this paper.

However, before describing typical guided wave nonreciprocal propagation and resonance experiments in plasmas and discussing their possible applications, it seems worthwhile to point out that, in general, the experimental conditions, for which results are presented below, justify the approximations of the theory with regard to the use of "quasi" TE_{11} waves in a plasma filled circular waveguide on a length of the order of 1–2 free space wavelength, even though rigorously the circular waves are not true modes of propagation in the plasma filled guide that is immersed in a longitudinal magnetic field.

We also would like to draw attention to the action of an applied magnetic field (whose intensity is varied) on the plasma itself in the absence of any electromagnetic energy.

[4] H. Suhl and L. R. Walker, *Bell. Sys. Tech. J.*, vol. 33, pp. 579, 939, and 1133; 1954.

[5] M. Bonnet, M. Matricon, and R. Roubine, *Ann. Telecommun.*, vol. 10, p. 150; 1955.

Experiments on Guided Microwave Propagation Through Magnetic Field Produced Anisotropic Gaseous Discharge Plasmas

In what follows we shall describe and illustrate experiments of Faraday rotation and gyroresonance in circular or other waveguide geometries with two different types of plasmas: 1) isothermal at room temperature and 2) dc or ac maintained nonisothermal plasmas. Both are under widely varied conditions of plasma frequencies and electron collision frequencies (nature and pressure of the gases in which the plasmas were produced). These experiments were carried out with microwaves in the C-band (5000 mc) and X-band (9000 mc) frequencies.

The waveguide Faraday effect consists of the rotation of the whole field pattern of the linearly polarized TE_{11} waves. The plane of the maximum electric field in this angularly dependent mode, which is along a diametral line, is taken as its plane of polarization. By rotation of the wave by an angle θ is meant a rotation by that angle of the whole field pattern and consequently that of the plane of polarization (maximum E field).

For $\theta = 90°$, linearly polarized TE_{11} waves are in independent states of polarization—there being a twofold degeneracy. Any TE_{11} wave in any state of polarization may be regarded as composed of two such independent TE_{11} component modes of appropriate amplitudes and phases. Circular polarization of a TE_{11} wave as well as linear polarization is therefore possible. Consequently, polarization transformations also can be produced.

For magneto-rotation experiments, a cylindrical gaseous discharge tube is located coaxially in a cylindrical waveguide in which TE_{11} waves are propagated at a low power level. The rf waves were either cw or pulsed. The tube lengths were of the order of 1–2 free space wavelengths completely immersed in a longitudinal magnetic field. The magnetic field for the C-band experiments was produced by solenoids surrounding the gas discharge tube filled portion of a cylindrical waveguide. The notations $\sigma = \omega_H/\omega$ and $q = \omega_p/\omega$ are used in the figures. The typical results below on magneto-rotation are relative to isothermal gaseous plasmas at room temperature ($\sim$300°K). These plasmas are obtained in the afterglow period of a decaying gaseous discharge. Fig. 2, opposite, shows the Faraday rotation of guided waves through an isothermal helium plasma at 300°K in a longitudinal magnetic field as a function of electron density. Fig. 3 illustrates the magneto-rotation of TE_{11} linear polarized waves through a neon plasma as a function of magnetic field for various gas pressures. It is apparent that an increase in the electron collision frequency (corresponding to an increase in gas pressure) reduces the amount of rotation obtainable with a given value of magnetic field. Fig. 4 is a comparison of the measured angle of rotation of a linear TE_{11} wave with the calculated angles obtained from direct measurements of the phase velocities of the two circularly polarized TE_{11} waves. As one would expect, the agreement is quite good, Fig. 5 and Fig. 6 are plots of the rotation angle vs magnetic field for various electron densities and gas pressures. Fig. 7 indicates the insertion loss for the two waves and illustrates how such an effect could be used as an isolator. Fig. 8 shows the ellipticity of the linear incident wave after passing through a plasma for various values of electron density or time after the discharge occurred. Fig. 9 demonstrates the variation of the TE_{11} wave amplitude which is reflected after it has traversed the plasma as measured by an antenna perpendicular to the launching antenna, as a function of the angle of rotation of the TE_{11} wave. This indicates the nonreciprocal properties of an anisotropic plasma when used as a propagation medium.

Resonance Phenomena in Gyromagnetic Plasmas at Microwave Frequencies

Since large rotation angles are obtained in the vicinity of a proper frequency of the anisotropic medium, it is of multiple interest to study the medium behavior for signal frequencies in the vicinity of these proper frequencies. The plasma frequency corresponding to each value of the surrounding magnetic field and the gyrofrequency of the electrons are such proper frequencies. This leads us to investigate the corresponding magneto-plasma and pure cyclotron resonances in the anisotropic plasmas.

The resonances for C- and X-band microwaves were studied in dc and pulsed (decaying) gaseous discharge plasmas permeated by dc or ac magnetic fields.

While neglecting electron collisional effects in plasmas in pure Faraday rotation for low level, rf waves can be justified in many practical cases, this can no longer be justified, however, when resonance conditions prevail in the gyroplasma. The electron collision frequency then becomes a controlling parameter, hence in the region of resonance all propagation effects become increasingly nonlinear. This implies that wave propagation under these conditions becomes very power level dependent (in these experiments even on the milliwatt power levels).

Indeed the width of the cyclotron resonance of the electrons is determined by $2\nu/\omega$. It is, therefore, a function of the nature of the gas, the pressure, the temperature of the electron gas consequently of the power level of the rf wave.

All other conditions being equal, the cyclotron resonance width will be in general broader in dc discharges than in lower electron temperature plasmas. This is illustrated in Fig. 11. Considering for instance the x component of the real part of the magneto-plasma conductivity (tensor)

$$\sigma_{rx} = \frac{nev_x}{E_x} = \frac{ne^2}{m\ \nu}\left[\frac{1 + j\omega/\nu}{1 + (\omega_H^2 - \omega^2)\dfrac{1}{\nu^2} + 2\dfrac{\omega}{\nu}}\right]. \quad (15)$$

Introducing the dimensionless quantities a, b, we have

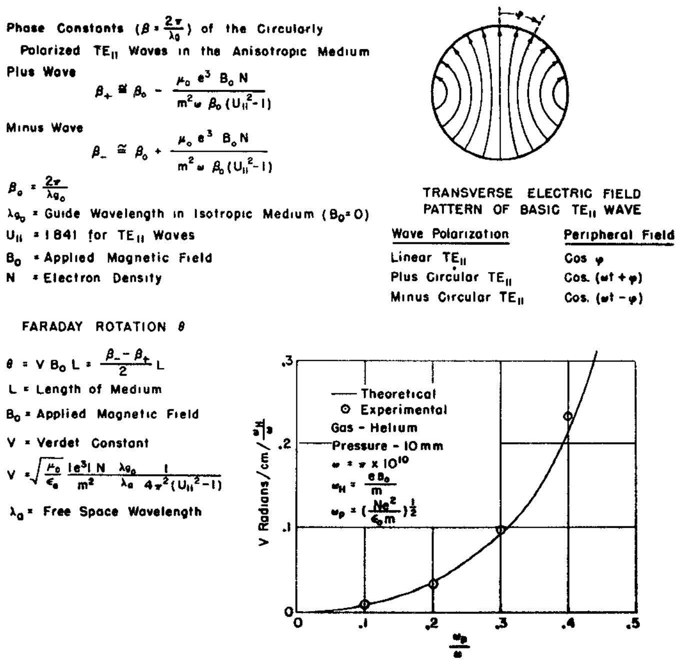


Fig. 2—Faraday rotation of guided waves.

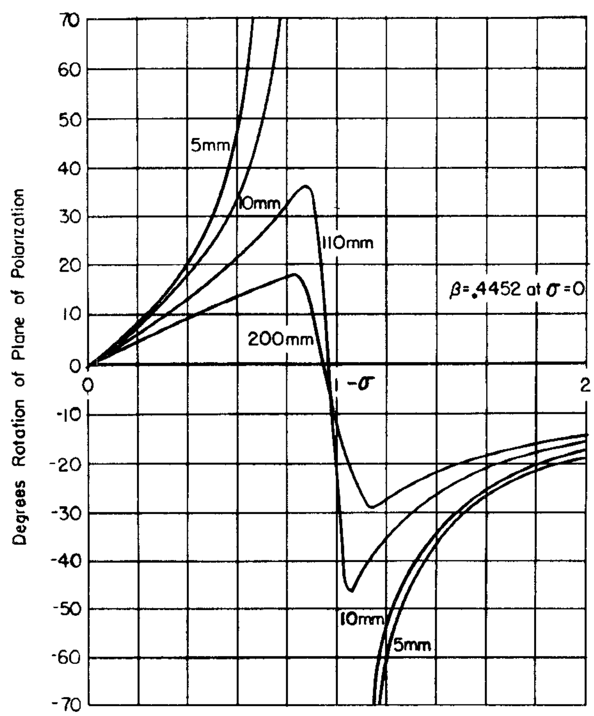


Fig. 3—Rotation of the polarization plane of the transmitted TE_{11} wave vs the magnetic field variable σ for several values of gas pressure. Gas-neon, frequency—5.10^9, discharge length—7.6 cm.

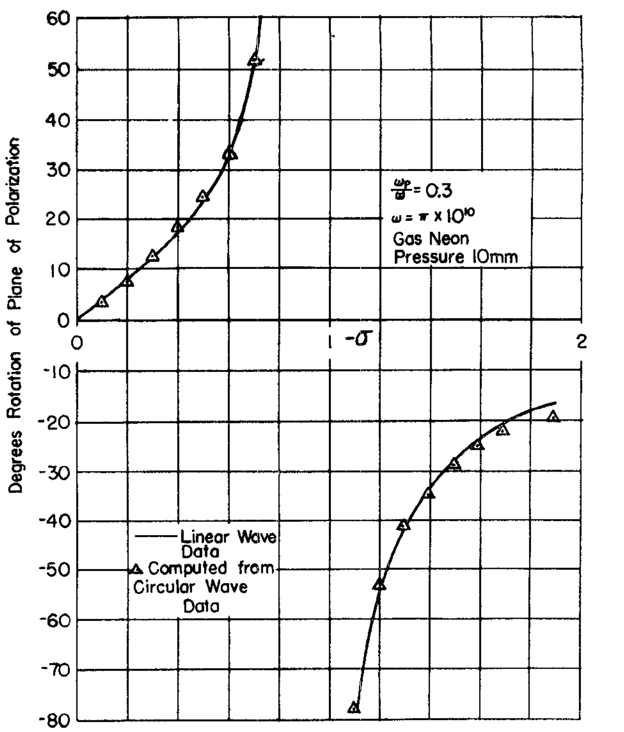


Fig. 4—Rotation of the polarization plane of the transmitted TE_{11} wave vs the magnetic field variable σ for a discharge 7.6-cm length.

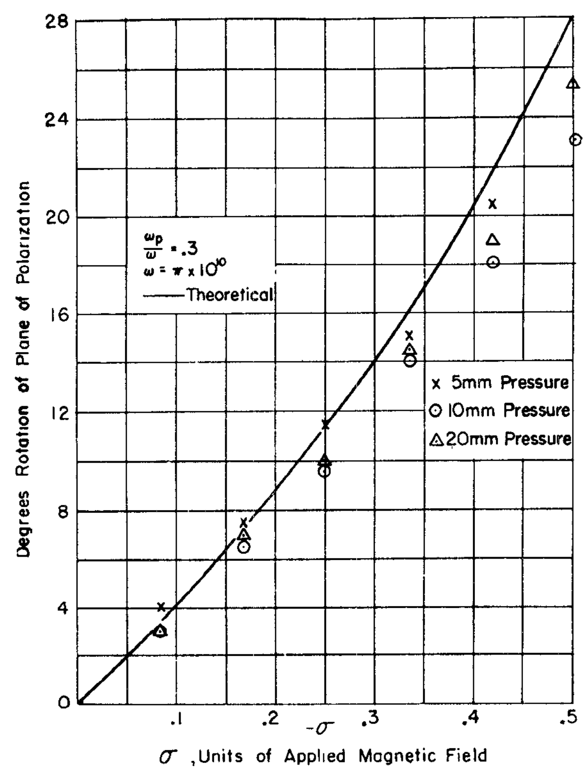


Fig. 5—Rotation of the plane of polarization of the transmitted TE_{11} wave for several values of pressure of a discharge in neon.

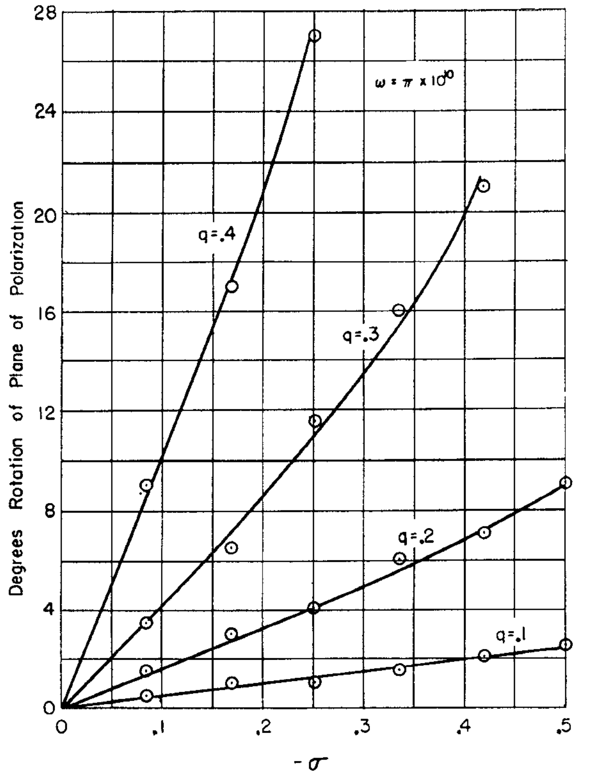


Fig. 6—Rotation of the plane of polarization of the transmitted TE_{11} wave for several values of the electron density variable q. Gas is helium at 10 mm Hg.

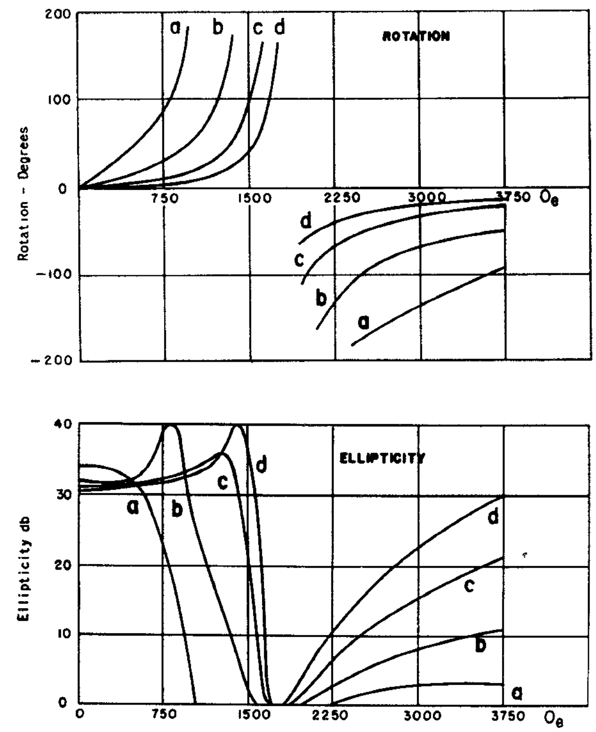


Fig. 8—Transmission of linearly polarized 5000-mc TE_{11} mode incident to decaying discharge. Neon gas, 8-mm Hg, delay (a) 200 μsec, (b) 400 μsec, (c) 700 μsec, (d) 1000 μsec.

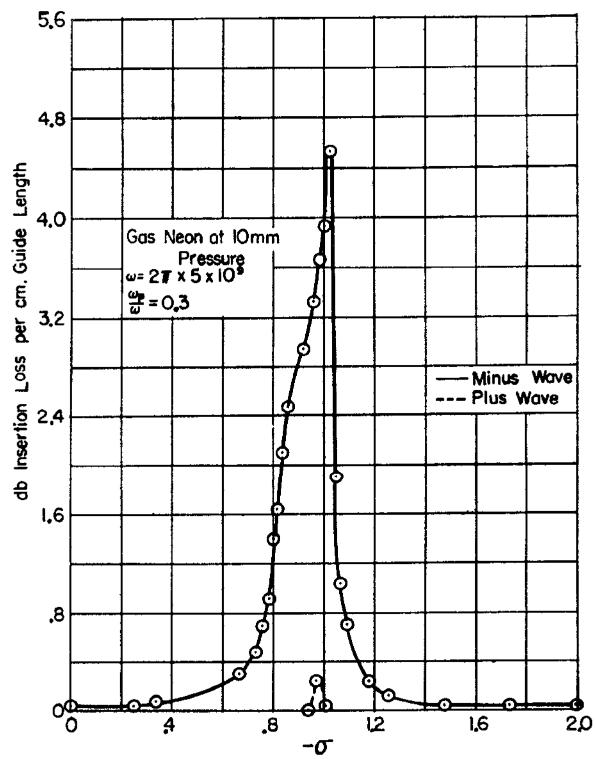


Fig. 7—Insertion loss of plus and minus waves as a function of applied magnetic field variable σ.

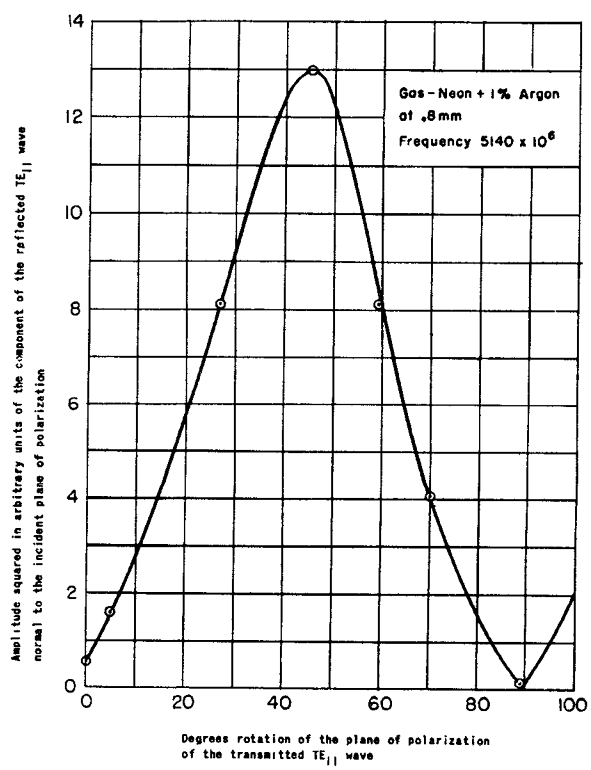


Fig. 9—Demonstration of the nonreciprocity of the anisotropic electron gas.

$$\frac{\sigma_{rx}}{\sigma_0} = \frac{1 + a^2 + b^2}{1 + b^2 - a^2 + 4a^2} \qquad (16)$$

where $a = \omega/\nu$ and $b = \omega_H/\nu$, and $ne^2/m\nu = \sigma_0$, the dc conductivity of the plasma.

For the extraordinary wave undergoing resonance, this is reduced to $\sigma_r/\sigma_0 = \frac{1}{2}$. This formula simplifies for the case of $\nu = \nu_{e-m} + \nu_{e-i}$ and can be written as

$$\sigma_{rx} = \frac{ne^2}{m(\nu_{e-m} + \nu_{ei})}.$$

In a gas pressure range above 1 mm Hg, in general, $\nu_{e-m} > \nu_{e-i}$. The controlling factor is the collision frequency of the electrons with neutral gas molecules. For very low gas pressures, (<1/10 mm Hg) and low-level (C-band) rf signals but adequately large n, $\nu_{e-m} < \nu_{e-i}$. Then for the extraordinary wave $\sigma_r = ne^2/m \cdot \nu_{e-i}$.

$$\nu_{e-i} = A \frac{n_i}{T_e^{3/2}} \log_n \left\{ \frac{B}{n_i^{1/2}} T_e \left(2 \frac{T_e \cdot T_i}{T_e + T_i} \right)^{1/2} \right\} \qquad (17)$$

neglecting the slowly varying logarithmic factor, $\nu_{e-i} \sim C(n_i/T_e^{3/2})$. Hence

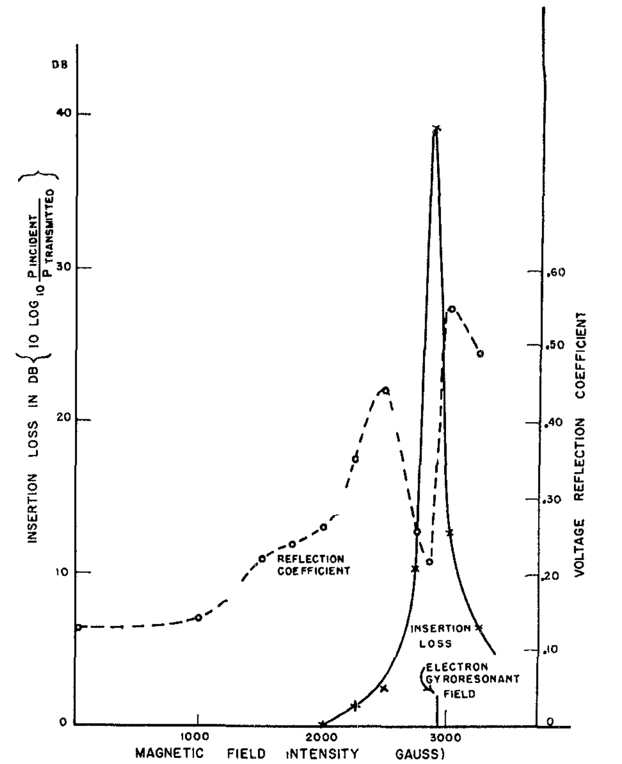
$$\sigma_r = \frac{1}{C} \frac{e^2}{m} \cdot \frac{n}{n_i} \cdot T_e^{3/2} \cong C' \cdot T_e^{3/2}.$$

Here A, B, C, and C' are constants, so that the resonance absorption for the above case is essentially independent of the number density of the charges and increases with increasing electron temperature, consequently with rf power level.

Resonance experiments have been performed in cylindrical, rectangular, and square waveguides. The gaseous discharge tubes, generally of glass, in the rectangular and square waveguides were either coaxial with the waveguide or perpendicular to the axis of the waveguide. A slowly varying low-intensity ac magnetic field (also longitudinal) was superimposed on the dc magnetic field and this was used in the detailed exploration of the resonances produced by the main dc magnetic field. Resonance experiments at X-band (8–10 kmc) frequencies made use of pulsed magnetic fields. Fig. 10 and Fig. 11 show resonance curves for dc discharge plasmas in X-band rectangular waveguide. Fig. 12 illustrates the tube geometry used in these experiments. Fig. 13 indicates the time sequence of the pulsed operations. Fig. 14 illustrates the magneto plasma and cyclotron resonances as a function of electron density and incident rf power. Fig. 15 demonstrates the effect on the plasma of the rf input power level. As can be seen on the first trace of each set, at a power level of 6 mw, the plasma properties change through resonance absorption heating of the electron gas.

Applications

The anisotropic gaseous plasma lends itself to applications, in low-level rf wave propagation control, simi-



Frequency: 8200 mc
Gas: Ne+1 per cent A at 0.1-mm Hg pressure
Discharge Conditions: Steady dc plasma at 465 v and 30 ma (current limiting resistor of 10,000 Ω included)
Length: L of solenoid = 5 inches ≃ 2 guide wavelengths

Fig. 10—Transmission and reflection characteristics vs magnetic field intensity.

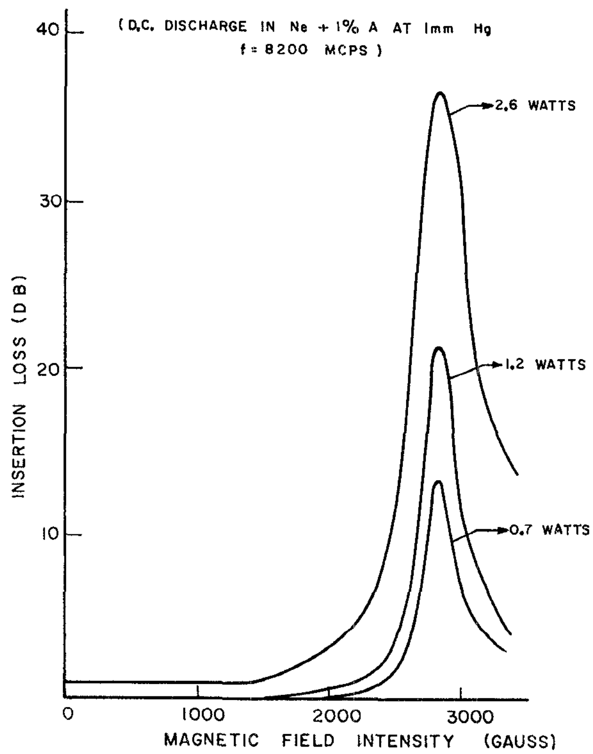


Fig. 11—Insertion loss vs magnetic field.

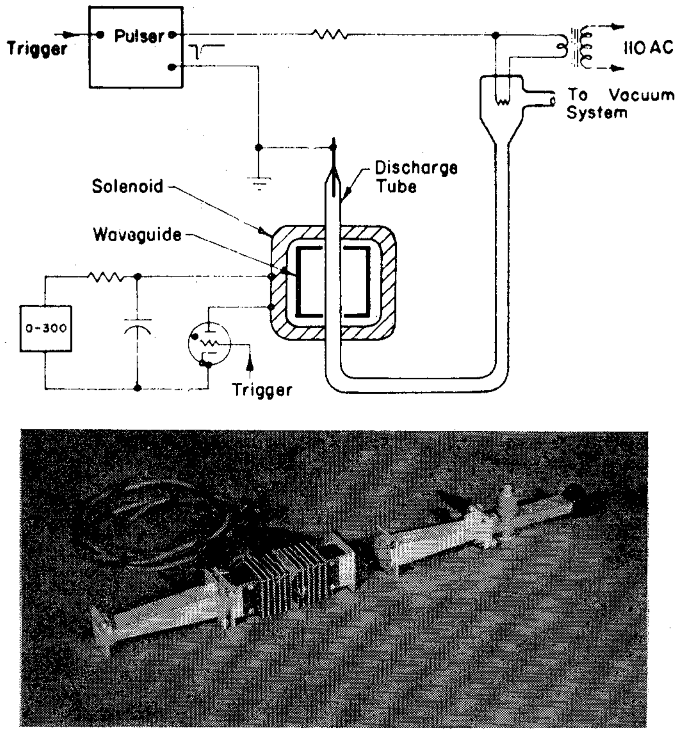


Fig. 12—Diagram of apparatus and photograph of waveguide and solenoid.

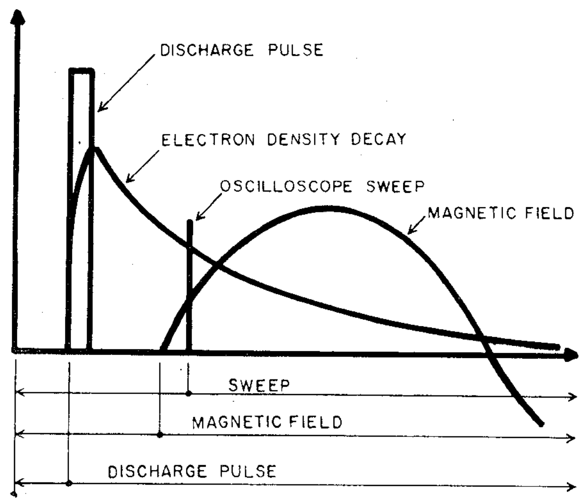


Fig. 13—Synchronization of apparatus.

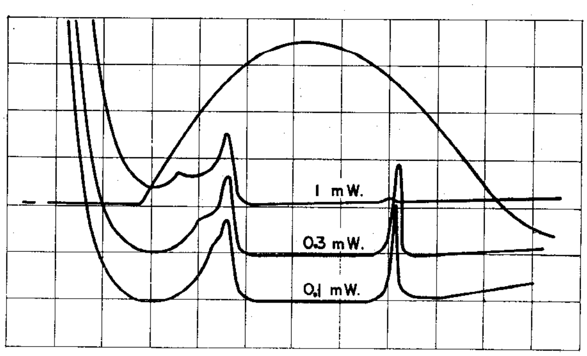
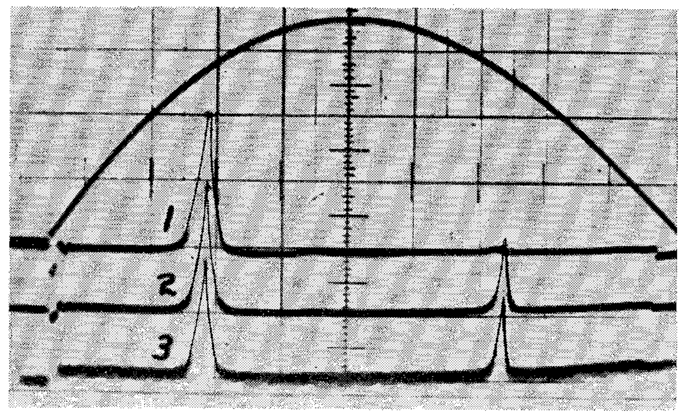


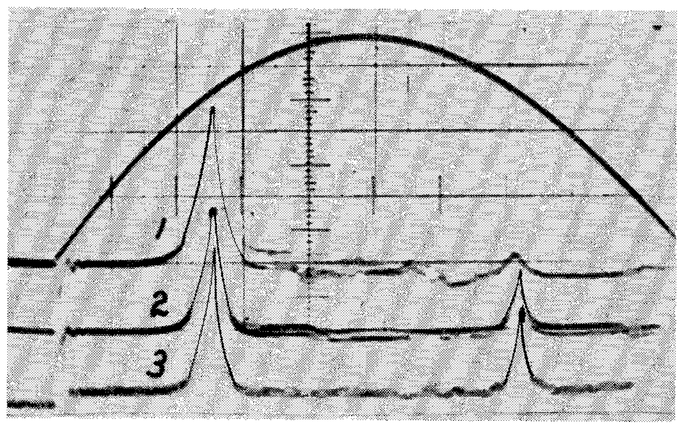
Fig. 14—Resonances—pulsed magnetic field.



(a)

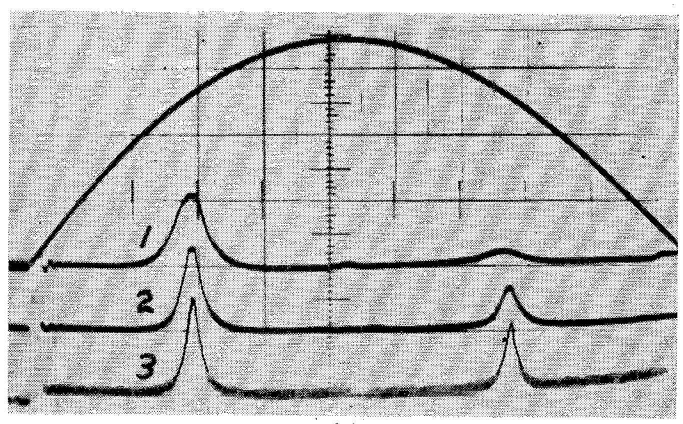
POWER LEVEL

1) 6 mw
2) 0.6 mw
3) 0.25 mw



(b)

1) 6 mw
2) 1 mw
3) 0.25 mw



(c)

1) 8 mw
2) 1.5 mw
3) 0.25 mw

Fig. 15—Resonances—pulsed magnetic field.
(a) 4-mm Hg, 175 μsec delay. Sweep Speed 40 μsec/div.
(b) 10-mm Hg, 200 μsec delay. Sweep Speed 30 μsec/div.
(c) 20-mm Hg, 350 μsec delay. Sweep Speed 30 μsec/div.

lar to those in which ferrites are used, with no limitations of frequencies.

For pulsed rf waves decaying gyroplasmas may be more advantageous than stationary media. With pulsed magnetic fields, isothermal gyroplasmas may be used, in particular, for rapid broad-band spectrum analysis. The limitation on the bandwidth is determined by the wave-guiding structure containing the magneto plasma. The fact that a gaseous discharge plasma can be established in any desired charge density state on microsecond or shorter time scale and be removed from that state on an equally short time scale makes the ionized gaseous medium a very flexible one whose potentialities have not been, as yet, explored to any great extent or even recognized by microwave engineers.

ACKNOWLEDGMENT

It is a pleasure to acknowledge here the contributions of M. A. Lampert, J. F. Heney, J. E. Etter, M. Gilden, L. P. McGrath, C. K. Chatterjee, and J. P. Monier who at one time or another have been associated with the author in the study of these problems.

The Three-Level Solid-State Maser*

H. E. D. SCOVIL†

Summary—**This article gives an introduction to amplification by solid-state maser techniques. Emphasis is placed on the three-level solid-state maser. The relevant physical properties of paramagnetic salts are discussed. The basis of the three-level excitation method is reviewed. Some design considerations are given. The design and performance characteristics of a particular device are mentioned.**

INTRODUCTION

MASERS (microwave amplifier by stimulated emission of radiation) offer the possibility of amplification with very low-noise figures. With suitable regeneration they may be converted into oscillators having a high degree of spectral purity.

The interaction medium consists of "uncharged" magnetic or electric dipoles. It is partially because of the lack of any charge fluctuations that these devices may exhibit low-noise characteristics. The medium is maintained in such a state that it presents negative loss or gain to incident radiation.

Beam-type masers,[1] because of their high stability, make excellent frequency standards. It is, however, just the properties that give them high stability, namely a high molecular Q and a fixed frequency, that limit their versatility as easily tunable broad-band amplifiers. In these respects solid-state masers offer advantages. The three-level maser now appears to be the most useful of the solid-state types since it amplifies in a continuous manner and its high permissible spin concentration leads to relatively large gain bandwidth products.

A review article on masers by Wittke has appeared.[2] It discusses the basic fundamentals as well as giving a brief description of each type. The purpose of the present article is to discuss in more detail the three-level solid-state maser.

The next section reviews some of the physical processes involved in the operation of the device. The following section discusses the three-level excitation method and the properties of suitable materials. Some remarks are then made about design considerations and the design and performance of a particular 6-kmc device is mentioned.

PROPERTIES AND PROCESSES OF THE MEDIUM

General Remarks

The maser medium consists of an ensemble of atomic magnetic moments or "spins" in the solid state. The individual dipoles may take up only certain discrete or "allowed" energy states as a result of interaction with crystalline electric and applied magnetic fields.

Since the medium chosen is such that the mutual interaction between dipoles is weak, the entire ensemble may be treated statistically as though all of the particles are distributed over the allowed states of an individual particle.

This system is referred to as the spin system. Interactions occur within the spin system and between the spin and the remainder of the crystal lattice as well as with a radiation field. These interactions are now discussed.

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[1] J. P. Gordon, H. J. Zeiger, and C. H. Townes, "The maser—new type of microwave amplifier, frequency standard, and spectrometer," *Phys. Rev.*, vol. 99, pp. 1264–1274; August, 1955.

[2] J. P. Wittke, "Molecular amplification and generation of microwaves," PROC. IRE, vol. 45, pp. 291–316; March, 1957.